COMMENT ON "TURBULENT CASCADES IN FOREIGN EXCHANGE MARKETS"

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Recently, Ghashghaie et al. have shown that some statistical aspects of fully developed turbulence and exchange rate fluctuations exhibit striking similarities [1]. The authors then suggested that the two problems might be deeply connected, and speculated on the existence of an 'information cascade' which would play the role in finance of the well known Kolmogorov energy cascade in turbulence [2].

Here we want to convince the reader that the two problems differ on a fundamental aspect, namely, correlations. Spatial correlations lead to the famous $-\frac{5}{3}$ power-law for the spectrum of the velocity fluctuations [2], cut-off on the low frequency side by the energy injection mechanism, and on the high frequency side by dissipation [1]. No temporal correlations of the sort are visible in the power-spectrum of financial time series — see Fig. 1, corresponding to the same data as studied in [1], where a slope of -2 is observed on a log-log scale. As seen in the inset, this corresponds to a totally 'white' spectrum for the price change signal (no correlations). If such correlations existed, by the way, it would be rather easy to use them to earn money!

The quality of the 'log-normal' fit reported in [1] can only be suggestive, since it is a two parameter fit *for each time delay* Δt. This must be compared to another proposal, which is that the fluctuations of financial assets are well described by a *truncated Lévy process*[3], or some other fat-tail process, with *independent* increments. This was originally suggested in [4] where the S&P 500 index was studied, and later substantiated on many other financial time series, using independent techniques (wavelets [5], or direct histogramming and convolution [M. Potters, unpublished data]). In Fig. 2, we show a 'truncated Lévy' fit of the data. There are three parameters, but which are once and for all fixed on the smallest time scale $\Delta t = 5$ min, while the larger time scales are obtained by convolution which amounts to assume independence of the increments. Notice that this method predicts both the shape and the overall scale of the distribution.

The mechanism by which fat tails in financial data disappear as the time scale grows is nothing but the consequence of the central limit theorem (see e.g. [6, 7]), which can safely be applied when these tails decay sufficiently fast and when correlations are absent. Subtle temporal correlations are actually observed in the evolution of the 'volatility', i.e. the amplitude of the fluctuations, leading to deviations from a simple convolution rule for the distribution of increments. However these deviations remain small and still allow for the application of the central limit theorem. On the other hand, the core of the problem in turbulence is precisely the existence of very strong correlations preventing the use of the central limit theorem!

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FIGURES

FIG. 1. Fourier transform of the price autocorrelation function $\langle x(t)x(t+\tau)\rangle$ as a function of temporal frequency for the DEM-USD exchange rate (Oct '91–Nov '94) (*bottom curve*) as compared to the $-\frac{5}{3}$ power-law spectrum observed for the spatial velocity fluctuations in turbulent flows (the data were recorded by Y. Gagne in a wind tunel experiment at $R_\lambda = 3050$) (*top curve*). Inset: Fourier transform of the price change autocorrelation function $\langle \Delta x(t)\Delta x(t{+}\tau)\rangle$, which is completely flat (and would behave as the $\frac{1}{3}^{\rm th}$ power of frequency in turbulent flows). Units are arbitrary.

FIG. 2. Cumulative distribution function of price changes for different time delays Δt =5 min, 15 min, 1 hour, 1 day and 5 days (from bottom to top). Symbols represent empirical data (DEM-USD, Oct '91–Nov '94): squares for positive price changes $(1 - F(\Delta x))$ and circles for negative ones $(F(-\Delta x))$. The solid line is a truncated symmetric Lévy distribution fitted to the data at 5 min and self-convoluted 3, 12, 84 and 420 times respectively. Note that the data used correspond to the 7 hour business day in New York.

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Comment on ‘‘Turbulent cascades...’’, Arneodo et al.
Figure 1

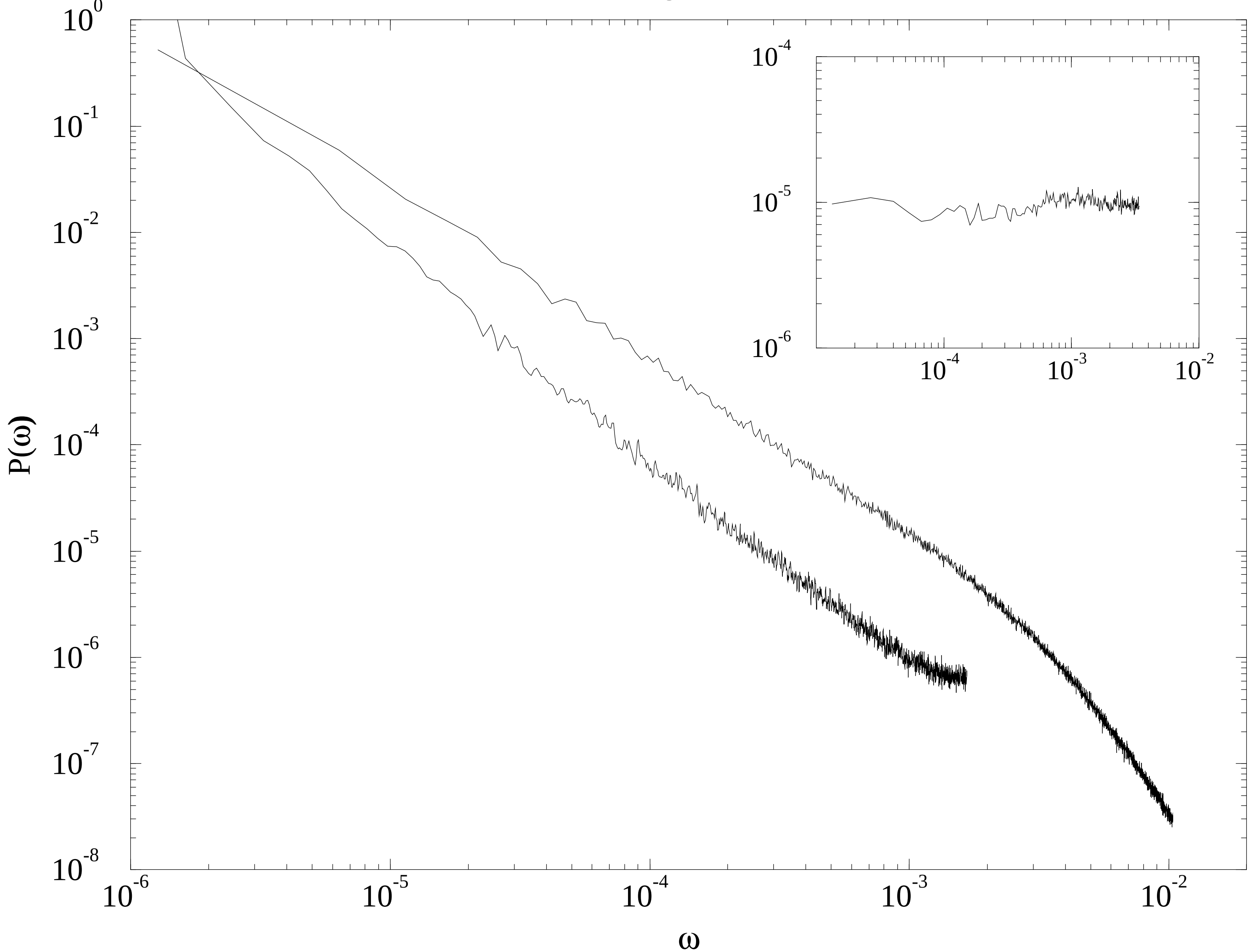

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Figure 2